

(f) (2)
結論

$$\left\{ \begin{array}{l} \cdot \text{Im } f_{E_{ij}} \text{ の次元} = 2(n-1) \\ \cdot \text{Ker } f_{E_{ij}} \text{ の次元} = n^2 - 2(n-1) \\ \quad = (n-1)^2 + 1 \end{array} \right.$$

基底 ① $i=j$ のとき

$$\text{Im } f_{E_{ii}} \text{ の基底} \quad \left\{ \begin{array}{l} E_{ik} \quad (k \neq i) \\ E_{ki} \quad (k \neq i) \end{array} \right.$$

$(n-1) \times 2$ 個

$$\text{Ker } f_{E_{ii}} \text{ の基底} \quad \left\{ \begin{array}{l} E_{kl} \quad (k \neq i \text{ かつ } l \neq i) \\ E_{ii} \end{array} \right.$$

$(n-1)^2 + 1$ 個

② $i \neq j$ のとき

$$\text{Ker } f_{E_{ij}} \text{ の基底} \quad \left\{ \begin{array}{l} E_{kl} \quad (k \neq j \text{ かつ } l \neq i) \\ E_{ii} + E_{jj} \end{array} \right.$$

$(n-1)^2 + 1$ 個

$$\text{Im } f_{E_{ij}} \text{ の基底} \quad \left\{ \begin{array}{l} E_{ik} \quad k \neq i, j \\ E_{kj} \quad k \neq i, j \\ E_{ij} \\ E_{ii} - E_{jj} \end{array} \right.$$

$(n-2) \times 2 + 1 + 1 = 2(n-1)$ 個

証明

① $v = \lambda qv$

②

$\ker f_{E_{k^c}} \ni E_{k^c}$

$(k^c \ni k^c)$

$\exists k^c$

③

$\text{Im } f_{E_{k^c}} \ni E_{k^c}$

(k^c)

$\exists k^c$

④

$\dim \ker f_{E_{k^c}} \geq (n-1)^2 + 1$

(1)

$\dim \text{Im } f_{E_{k^c}} \geq 2(n-1)$

(2)

$\therefore \dim M(n, \mathbb{R}) = \dim \ker f_{E_{k^c}} + \dim \text{Im } f_{E_{k^c}} \geq (n-1)^2 + 1 + 2(n-1) = n^2$

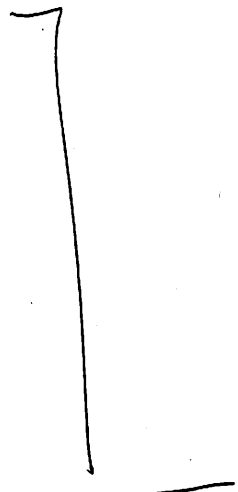
(3)

$= n^2$

③の不等式が等号成立

①(2)の不等式が等号成立



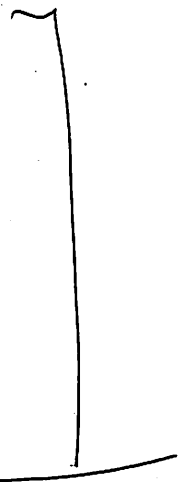


例 3

① $c = 1$ のとき

$$\text{Im } f_{E_{cc}} = \left\langle \begin{matrix} E_{kk} \\ E_{lk} \end{matrix} \right\rangle = \left\langle \begin{matrix} (k \neq l) \\ (k \neq l) \end{matrix} \right\rangle$$

$$\begin{matrix} \supset \\ \supset \\ \supset \end{matrix}$$



例 2

① $c = 1$ のとき

$$\text{Ker } f_{E_{cc}} = \left\langle \begin{matrix} E_{kk} \\ E_{ll} \end{matrix} \right\rangle = \left\langle (k \neq l) \right\rangle$$

$$\begin{matrix} \supset \\ \supset \\ \supset \end{matrix}$$