$\boxed{3}$

(c) Bの固有多項式 $= (t+10)^2 (t+20)^2$

$$W(B;-10) = \left\langle \begin{pmatrix} 1-2i \\ -1 \\ 2 \end{pmatrix} \right\rangle$$

$$W(B,-20) = \left\langle \begin{pmatrix} \frac{1-2i}{5} \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{-2+4i}{5} \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

Cの固有多項式 $= (t-10)^2(t-20)$

$$W(C;10) = \left\langle \begin{pmatrix} -\frac{1-2i}{5} \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{2-4i}{5} \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

$$W(C;20) = \left\langle \begin{pmatrix} -1+2i \\ -1 \\ 2 \end{pmatrix} \right\rangle$$

(d)

	$W(C,10)$	$W(C,20)$
$W(B,-10)$	$\left\langle \begin{pmatrix} \frac{1-2i}{2} \\ -\frac{1}{2} \\ 1 \end{pmatrix} \right\rangle$	$\{0\}$
$W(B,-20)$	$\left\langle \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \right\rangle$	$\left\langle \begin{pmatrix} \frac{-1+2i}{2} \\ -\frac{1}{2} \\ 1 \end{pmatrix} \right\rangle$

(e) $\frac{1}{\sqrt{10}}\begin{pmatrix} 1-2i \\ -1 \\ 2 \end{pmatrix}, \frac{1}{\sqrt{10}}\begin{pmatrix} -1+2i \\ -1 \\ 2 \end{pmatrix}, \frac{1}{\sqrt{5}}\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$

(g) $$U = \begin{pmatrix} \frac{1-2i}{\sqrt{10}} & \frac{-1+2i}{\sqrt{10}} & 0 \\ -\frac{1}{\sqrt{10}} & -\frac{1}{\sqrt{10}} & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{10}} & \frac{2}{\sqrt{10}} & \frac{1}{\sqrt{5}} \end{pmatrix}$$

$$U^* B U = \begin{pmatrix} -10 & & \\ & -20 & \\ & & -20 \end{pmatrix}$$

$$U^* C U = \begin{pmatrix} 10 & & \\ & 10 & \\ & & 20 \end{pmatrix}$$

$$U^* A U = \begin{pmatrix} -10+10i & & \\ & -20+10i & \\ & & -20+20i \end{pmatrix}$$

$\boxed{1}$ Aの固有多項式

$$= (t+10-10i)(t+20-10i)(t+20-20i)$$

固有空間

$$W(-10+10i, A) = \left\langle \begin{pmatrix} \frac{1-2i}{2} \\ -\frac{1}{2} \\ 1 \end{pmatrix} \right\rangle$$

$$W(-20+10i, A) = \left\langle \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \right\rangle$$

$$W(-20+20i, A) = \left\langle \begin{pmatrix} \frac{-1+2i}{2} \\ -\frac{1}{2} \\ 1 \end{pmatrix} \right\rangle$$

以下は$\boxed{3}$の(e)(g)と同じ流れ。

$\boxed{2}$

$$B = \frac{1}{2}(A + A^*)$$

$$C = \frac{1}{2i}(A - A^*)$$

(a) $(A^*)^* = A$ を使う

$$B^* = \frac{1}{2}(A + A^*)^*$$

$$= \frac{1}{2}(A^* + (A^*)^*)$$

$$= \frac{1}{2}(A^* + A) = B.$$

$$C^* = \left(\frac{1}{2i}(A - A^*)\right)^*$$

$$= -\frac{1}{2i}(A^* - A^{**})$$

$$= -\frac{1}{2i}(A^* - A)$$

$$= \frac{1}{2i}(A - A^*)$$

$$= C$$

(b)

$$\begin{cases} 2B = A + A^* \\ 2iC = A - A^* \end{cases}$$

$$\begin{cases} 2B + 2iC = 2A \\ 2B - 2iC = 2A^* \end{cases}$$

$$A = B + iC$$

$$A^* = B - iC$$

(d)(e) $$AA^* = (B+iC)(B-iC)$$

$$= B^2 + C^2 + i(CB - BC)$$

$$A^*A = (B-iC)(B+iC)$$

$$= B^2 + C^2 - i(CB - BC)$$

A：正規 $\Leftrightarrow AA^* = A^*A$

$\Leftrightarrow CB - BC = 0$

$\Leftrightarrow BC = CB$.

(f)

$$B\mathbb{v} = \lambda \mathbb{v}$$

$$\begin{array}{ccc} CB\mathbb{v} & = & C\lambda\mathbb{v} \\ \shortparallel & & \shortparallel \\ BC\mathbb{v} & & \lambda C\mathbb{v} \end{array}$$

$C\mathbb{v} \neq 0$ ならば $C\mathbb{v}$ は B の固有値λの固有ベクトルである.